Chapter 2 Motion and Recombination of Electrons and Holes

2.1 Thermal Motion

Average electron or hole kinetic energy

\[ \frac{1}{2} kT = \frac{1}{2} m v_{th}^2 \]

\[ v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ JK}^{-1} \times 300 \text{ K}}{0.26 \times 9.1 \times 10^{-31} \text{ kg}}} \]

\[ = 2.3 \times 10^5 \text{ m/s} = 2.3 \times 10^7 \text{ cm/s} \]
2.1 Thermal Motion

- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero.
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$
Hot-point Probe can determine sample doing type

*Thermoelectric Generator*(from heat to electricity) and *Cooler*(from electricity to refrigeration)
2.2 Drift

2.2.1 Electron and Hole Mobilities

- Drift is the motion caused by an electric field.
2.2.1 Electron and Hole Mobilities

\[ m_p \nu = qE \tau_{mp} \]

\[ \nu = \frac{qE \tau_{mp}}{m_p} \]

\[ \nu = \mu_p E \]

\[ \mu_p = \frac{q \tau_{mp}}{m_p} \]

\[ \nu = -\mu_n E \]

\[ \mu_n = \frac{q \tau_{mn}}{m_n} \]

- \( \mu_p \) is the hole mobility and \( \mu_n \) is the electron mobility
2.2.1 Electron and Hole Mobilities

\[ \nu = \mu E \; ; \; \mu \text{ has the dimensions of } \frac{\nu}{E} \left[ \frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right]. \]

**Electron and hole mobilities of selected semiconductors**

<table>
<thead>
<tr>
<th></th>
<th>Si</th>
<th>Ge</th>
<th>GaAs</th>
<th>InAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_n ) (cm(^2)/V( \cdot )s)</td>
<td>1400</td>
<td>3900</td>
<td>8500</td>
<td>30000</td>
</tr>
<tr>
<td>( \mu_p ) (cm(^2)/V( \cdot )s)</td>
<td>470</td>
<td>1900</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
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Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?
Drift Velocity, Mean Free Time, Mean Free Path

**EXAMPLE:** Given \( \mu_p = 470 \text{ cm}^2/\text{V} \cdot \text{s} \), what is the hole drift velocity at \( E = 10^3 \text{ V/cm} \)? What is \( \tau_{mp} \) and what is the distance traveled between collisions (called the **mean free path**)? Hint: When in doubt, use the MKS system of units.

**Solution:**

\[
\nu = \mu_p E = 470 \text{ cm}^2/\text{V} \cdot \text{s} \times 10^3 \text{ V/cm} = 4.7 \times 10^5 \text{ cm/s}
\]

\[
\tau_{mp} = \frac{\mu_p m_p}{q} = 470 \text{ cm}^2/\text{V} \cdot \text{s} \times 0.39 \times 9.1 \times 10^{-31} \text{ kg}/1.6 \times 10^{-19} \text{ C}
\]

\[
= 0.047 \text{ m}^2/\text{V} \cdot \text{s} \times 2.2 \times 10^{-12} \text{ kg/C} = 1 \times 10^{-13} \text{ s} = 0.1 \text{ ps}
\]

\[
\text{mean free path} = \tau_{mh} \nu_{th} \sim 1 \times 10^{-13} \text{ s} \times 2.2 \times 10^7 \text{ cm/s}
\]

\[
= 2.2 \times 10^{-6} \text{ cm} = 220 \text{ Å} = 22 \text{ nm}
\]

This is smaller than the typical dimensions of devices, but getting close.
2.2.2 Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:
1. Phonon Scattering
2. Ionized-Impurity (Coulombic) Scattering

**Phonon scattering** mobility decreases when temperature rises:

\[
\mu_{\text{phonon}} \propto \tau_{\text{phonon}} \propto \frac{1}{\text{phonon density} \times \text{carrier thermal velocity}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}
\]

\[
\mu = q \tau / m \propto T
\]

\[
v_{\text{th}} \propto T^{1/2}
\]
Impurity (Dopant)-Ion Scattering or Coulombic Scattering

There is less change in the direction of travel if the electron zips by the ion at a higher speed.

\[ \mu_{impurity} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d} \]
Total Mobility

\[
\frac{1}{\tau} = \frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{impurity}}}
\]

\[
\frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}}
\]

Electrons

Holes

Mobility (cm\(^2\) V\(^{-1}\) s\(^{-1}\))

Total Impurity Concentration (atoms cm\(^{-3}\))

[Image of a graph showing the relationship between mobility and total impurity concentration for both electrons and holes.]
Temperature Effect on Mobility

Question:
What \( N_d \) will make \( d\mu_n/dT = 0 \) at room temperature?
Velocity Saturation

• When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy.

• Therefore, the kinetic energy is capped at large $E$, and the velocity does not rise above a saturation velocity, $v_{sat}$.

• Velocity saturation has a deleterious effect on device speed as shown in Ch. 6.
2.2.3 Drift Current and Conductivity

Hole current density \( J_p = qp\nu \) A/cm\(^2\) or C/cm\(^2\)·sec

**EXAMPLE:** If \( p = 10^{15}\) cm\(^{-3}\) and \( \nu = 10^{4}\) cm/s, then

\[
J_p = 1.6 \times 10^{-19} \text{C} \times 10^{15} \text{cm}^{-3} \times 10^{4}\text{cm/s} \\
= 1.6 \text{C/s} \cdot \text{cm}^2 = 1.6 \text{A/cm}^2
\]
2.2.3 Drift Current and Conductivity

\[ J_{p,\text{drift}} = q_p \nu = q_p \mu_p E \]

\[ J_{n,\text{drift}} = -q_n \nu = q_n \mu_n E \]

\[ J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = \sigma E = (q_n \mu_n + q_p \mu_p)E \]

\[ \therefore \text{conductivity} \ (1/\text{ohm-cm}) \text{ of a semiconductor is} \]
\[ \sigma = q_n \mu_n + q_p \mu_p \]

\[ 1/\sigma = \text{is resistivity (ohm-cm)} \]
Relationship between Resistivity and Dopant Density

\[ \rho = \frac{1}{\sigma} \]

- **P-type**
- **N-type**

DOPANT DENSITY cm\(^{-3}\)

RESISTIVITY (\(\Omega\cdot\text{cm}\))
EXAMPLE: Temperature Dependence of Resistance

(a) What is the resistivity ($\rho$) of silicon doped with $10^{17} \text{cm}^{-3}$ of arsenic?

(b) What is the resistance ($R$) of a piece of this silicon material 1 $\mu$m long and 0.1 $\mu$m$^2$ in cross-sectional area?

Solution:

(a) Using the N-type curve in the previous figure, we find that $\rho = 0.084 \ \Omega\text{-cm}$.

(b) $R = \rho L/A = 0.084 \ \Omega\text{-cm} \times 1 \ \mu\text{m} / 0.1 \ \mu\text{m}^2$

$= 0.084 \ \Omega\text{-cm} \times 10^{-4} \ \text{cm} / 10^{-10} \ \text{cm}^2$

$= 8.4 \times 10^{-4} \ \Omega$
EXAMPLE: Temperature Dependence of Resistance

By what factor will $R$ increase or decrease from $T=300$ K to $T=400$ K?

Solution: The temperature dependent factor in $\sigma$ (and therefore $\rho$) is $\mu_n$. From the mobility vs. temperature curve for $10^{17}$ cm$^{-3}$, we find that $\mu_n$ decreases from 770 at 300K to 400 at 400K. As a result, $R$ increases by

$$\frac{770}{400} = 1.93$$
2.3 Diffusion Current

Particles diffuse from a higher-concentration location to a lower-concentration location.
2.3 Diffusion Current

\[ J_{n,\text{diffusion}} = qD_n \frac{dn}{dx} \quad J_{p,\text{diffusion}} = -qD_p \frac{dp}{dx} \]

\( D \) is called the diffusion constant. Signs explained:
Total Current – Review of Four Current Components

\[ J_{TOTAL} = J_n + J_p \]

\[ J_n = J_{n,drift} + J_{n,diffusion} = qn \mu_n E + qD_n \frac{dn}{dx} \]

\[ J_p = J_{p,drift} + J_{p,diffusion} = qp \mu_p E - qD_p \frac{dp}{dx} \]
2.4 Relation Between the Energy Diagram and $V$, $E$

$E_c$ and $E_v$ vary in the opposite direction from the voltage. That is, $E_c$ and $E_v$ are higher where the voltage is lower.

\[
E(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}
\]
2.5 Einstein Relationship between $D$ and $\mu$

Consider a piece of non-uniformly doped semiconductor.

**N-type semiconductor**

- Decreasing donor concentration

\[
n = N_c e^{-\frac{(E_c - E_f)}{kT}}
\]

\[
\frac{dn}{dx} = -\frac{N_c}{kT} e^{-\frac{(E_c - E_f)}{kT}} \frac{dE_c}{dx}
\]

\[
E_c(x) = -\frac{n}{kT} \frac{dE_c}{dx}
\]

\[
= -\frac{n}{kT} qE
\]
2.5 Einstein Relationship between $D$ and $\mu$

\[ \frac{dn}{dx} = -\frac{n}{kT} qE \]

\[ J_n = qn\mu_n E + qD_n \frac{dn}{dx} = 0 \] at equilibrium.

\[ 0 = qn\mu_n E - qn \frac{qD_n}{kT} E \]

\[ D_n = \frac{kT}{q} \mu_n \quad \text{Similarly}, \quad D_p = \frac{kT}{q} \mu_p \]

These are known as the Einstein relationship.
**EXAMPLE: Diffusion Constant**

What is the hole diffusion constant in a piece of silicon with $\mu_p = 410 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$?

**Solution:**

$$D_p = \left( \frac{kT}{q} \right) \mu_p = (26 \text{ mV}) \cdot 410 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1} = 11 \text{ cm}^2 / \text{s}$$

**Remember:** $kT/q = 26 \text{ mV at room temperature}$. 
2.6 Electron-Hole Recombination

- The equilibrium carrier concentrations are denoted with $n_0$ and $p_0$.
- The total electron and hole concentrations can be different from $n_0$ and $p_0$.
- The differences are called the *excess carrier concentrations* $n'$ and $p'$.

\[
\begin{align*}
n &= n_0 + n' \\
p &= p_0 + p'
\end{align*}
\]
Charge Neutrality

- Charge neutrality is satisfied at equilibrium ($n' = p' = 0$).
- When a non-zero $n'$ is present, an equal $p'$ may be assumed to be present to maintain charge equality and vice-versa.
- If charge neutrality is not satisfied, the net charge will attract or repel the (majority) carriers through the drift current until neutrality is restored.

\[ n' = p' \]
Recombination Lifetime

• Assume light generates $n'$ and $p'$. If the light is suddenly turned off, $n'$ and $p'$ decay with time until they become zero.
• The process of decay is called recombination.
• The time constant of decay is the recombination time or carrier lifetime, $\tau$.
• Recombination is nature’s way of restoring equilibrium ($n' = p' = 0$).
Recombination Lifetime

- \( \tau \) ranges from 1ns to 1ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt.
- These *deep traps* capture electrons and holes to facilitate recombination and are called *recombination centers*.

![Diagram showing the energy levels and recombination process](image)

- Direct Recombination is unfavorable in silicon.
- Recombination centers are indicated by the deep trap levels.
Direct and Indirect Band Gap

Direct band gap
Example: GaAs

Direct recombination is efficient as $k$ conservation is satisfied.

Indirect band gap
Example: Si

Direct recombination is rare as $k$ conservation is not satisfied.

Modern Semiconductor Devices for Integrated Circuits (C. Hu)
Rate of recombination \((s^{-1} cm^{-3})\)

\[
\frac{dn'}{dt} = -\frac{n'}{\tau}
\]

\[
n' = p'
\]

\[
\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} = \frac{dp'}{dt}
\]
EXAMPLE: Photoconductors

A bar of Si is doped with boron at $10^{15}\text{cm}^{-3}$. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of $10^{20}/\text{s}\cdot\text{cm}^3$. The recombination lifetime is $10\mu\text{s}$. What are (a) $p_0$, (b) $n_0$, (c) $p'$, (d) $n'$, (e) $p$, (f) $n$, and (g) the $np$ product?
EXAMPLE: Photoconductors

Solution:

(a) What is $p_0$?

$$p_0 = N_a = 10^{15} \text{ cm}^{-3}$$

(b) What is $n_0$?

$$n_0 = n_i^2/p_0 = 10^5 \text{ cm}^{-3}$$

(c) What is $p'$?

In steady-state, the rate of generation is equal to the rate of recombination.

$$10^{20}/\text{s-cm}^3 = p'/\tau$$

$$\therefore p' = 10^{20}/\text{s-cm}^3 \cdot 10^{-5}\text{s} = 10^{15} \text{ cm}^{-3}$$
EXAMPLE: Photoconductors

(d) What is $n'$?
\[ n' = p' = 10^{15} \text{ cm}^{-3} \]

(e) What is $p$?
\[ p = p_0 + p' = 10^{15} \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} = 2 \times 10^{15} \text{ cm}^{-3} \]

(f) What is $n$?
\[ n = n_0 + n' = 10^5 \text{ cm}^{-3} + 10^{15} \text{ cm}^{-3} \sim 10^{15} \text{ cm}^{-3} \text{ since } n_0 << n' \]

(g) What is $np$?
\[ np \sim 2 \times 10^{15} \text{ cm}^{-3} \cdot 10^{15} \text{ cm}^{-3} = 2 \times 10^{30} \text{ cm}^{-6} \gg n_i^2 = 10^{20} \text{ cm}^{-6}. \]
\[ \text{The } np \text{ product can be very different from } n_i^2. \]
2.7 Thermal Generation

If $n'$ is negative, there are fewer electrons than the equilibrium value.

As a result, there is a net rate of \textit{thermal generation} at the rate of $|n'|/\tau$. 
2.8 Quasi-equilibrium and Quasi-Fermi Levels

- Whenever $n' = p' \neq 0$, $np \neq n_i^2$. We would like to preserve and use the simple relations:
  \[
  n = N_c e^{-\left(E_c - E_f\right)/kT} \\
  p = N_v e^{-\left(E_f - E_v\right)/kT}
  \]

- But these equations lead to $np = n_i^2$. The solution is to introduce two quasi-Fermi levels $E_{fn}$ and $E_{fp}$ such that
  \[
  n = N_c e^{-\left(E_c - E_{fn}\right)/kT} \\
  p = N_v e^{-\left(E_{fp} - E_v\right)/kT}
  \]

Even when electrons and holes are not at equilibrium, within each group the carriers can be at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.
EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Consider a Si sample with $N_d=10^{17}\text{cm}^{-3}$ and $n'=p'=10^{15}\text{cm}^{-3}$.

(a) Find $E_f$.

$$n = N_d = 10^{17}\text{cm}^{-3} = N_c \exp\left[-\frac{(E_c - E_f)}{kT}\right]$$

$$\therefore E_c - E_f = 0.15\text{ eV}. \quad (E_f \ \text{is below} \ E_c \ \text{by} \ 0.15\text{ eV}.)$$

Note: $n'$ and $p'$ are much less than the majority carrier concentration. This condition is called low-level injection.
EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Now assume \( n' = p' = 10^{15} \text{ cm}^{-3} \).

(b) Find \( E_{fn} \) and \( E_{fp} \).

\[
\begin{align*}
n &= 1.01 \times 10^{17} \text{ cm}^{-3} = N_c e^{-(E_c - E_{fn})/kT} \\
\therefore \quad E_c - E_{fn} &= kT \times \ln(N_c / 1.01 \times 10^{17} \text{ cm}^{-3}) \\
&= 26 \text{ meV} \times \ln(2.8 \times 10^{19} \text{ cm}^{-3} / 1.01 \times 10^{17} \text{ cm}^{-3}) \\
&= 0.15 \text{ eV}
\end{align*}
\]

\( E_{fn} \) is nearly identical to \( E_f \) because \( n \approx n_0 \).
EXAMPLE: Quasi-Fermi Levels

\[ p = 10^{15} \text{ cm}^{-3} = N_v e^{-(E_{fp} - E_v)/kT} \]

\[ \therefore E_{fp} - E_v = kT \times \ln(N_v/10^{15} \text{ cm}^{-3}) \]
\[ = 26 \text{ meV} \times \ln(1.04 \times 10^{19} \text{ cm}^{-3}/10^{15} \text{ cm}^{-3}) \]
\[ = 0.24 \text{ eV} \]
2.9 Chapter Summary

\( v_p = \mu_p E \)

\( v_n = -\mu_n E \)

\( J_{p,drift} = q p \mu_p E \)

\( J_{n,drift} = q n \mu_n E \)

\( J_{n,diffusion} = q D_n \frac{dn}{dx} \)

\( J_{p,diffusion} = -q D_p \frac{dp}{dx} \)

\( D_n = \frac{kT}{q} \mu_n \)

\( D_p = \frac{kT}{q} \mu_p \)
2.9 Chapter Summary

\( \tau \) is the recombination lifetime.

\( n' \) and \( p' \) are the excess carrier concentrations.

\[
\begin{align*}
n &= n_0 + n' \\
p &= p_0 + p'
\end{align*}
\]

Charge neutrality requires \( n' = p' \).

\[
rate \ of \ recombination = \frac{n'}{\tau} = \frac{p'}{\tau}
\]

\( E_{fn} \) and \( E_{fp} \) are the quasi-Fermi levels of electrons and holes.

\[
\begin{align*}
n &= N_c e^{-(E_c - E_{fn})/kT} \\
p &= N_v e^{-(E_{fp} - E_v)/kT}
\end{align*}
\]